ELEC 3106
Study Notes

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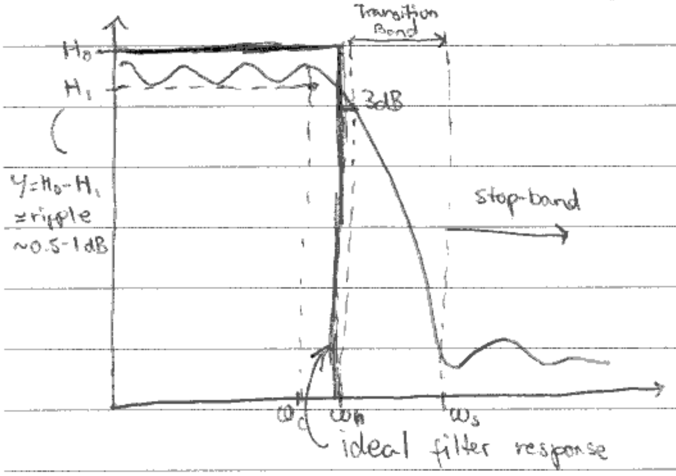
NOTES: Week 10 - Active Filters

Of course, it is not possible to get perfect filters. More like this:



ω_h = 3dB bandwidth

ω_s = min attenuation frequency.

- the pass band is transmitted without excessive attenuation.
- aim for perfect falloff. (higher order filters)
- ω_c = ripple bandwidth.

Filter Specification

i, range of passband frequencies (ω_H : LPF)

ii, stopband attenuation ($H_0 - H_2$)

iii, stopband frequency range

iv, allowable passband ripple $\gamma = H_0 - H_1$

v, other factors including impedance, phase & transient response.

Analogue network function: $H(s) = \dfrac{N(s)}{D(s)}$

Attenuation of the network defined as $A(\omega) = -20\log|H(j\omega)|$

Maximally Flat Approximation

$$|H(j\omega)|^2 = \frac{H_0^2}{1 + \varepsilon^2\left(\frac{\omega}{\omega_0}\right)^{2n}}$$

→ The first 2 derivatives are 0.
ε = ripple factor.

ε & n controll the pass band loss. Attenuation is hence:

$$A(\omega) = 10\log\left[1 + \varepsilon^2\left(\frac{\omega}{\omega_0}\right)^{2n}\right]$$

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- There exists a trade off between accuracy and complexity when choosing order.

$$\epsilon = \left(\frac{\omega_0}{\omega}\right)^n \left(10^{0.1A(\omega)} - 1\right)^{\frac{1}{2}}$$

- Suppose max. passband loss allowed occurs at $\omega = \omega_0$ & $A(\omega) = A_p$, then

$$\epsilon = \left(10^{0.1A_p} - 1\right)^{\frac{1}{2}} \qquad (\omega = \omega_0)$$

level of attenuation in dB

- Suppose minimum stop band attenuation at ω_s is A_a, then

$$n \geq \frac{\log\left[\left(10^{0.1A_a} - 1\right) / \left(10^{0.1A_p} - 1\right)\right]}{2\log(\omega_s/\omega_0)}$$

We choose the least integer value of n that satisfies the inequality.

- SPECIAL CASE, $\epsilon = 1$ in a maximally flat filter we will get the Butterworth polynomials. A Butterworth filter is a sub-class of the max flat filter.
  - ↳ $A_p = 3\text{dB}$ at $\omega = \omega_0$.
  - ↳ $|H(j\omega)|^2 = \dfrac{H_0}{1 + \left(\frac{\omega}{\omega_0}\right)^{2n}} = \dfrac{H_0}{B_n^2(\omega)}$

If you design a Butterworth filter, if n is even, the products are of quadratic factors. If n is odd, product contains $(s+1) \times$ quadratic.

Equiripple Approximation (Chebychev)

$$|H(j\omega)|^2 = \frac{H_0^2}{1 + \epsilon^2 C_n^2\left(\frac{\omega}{\omega_c}\right)}$$

passband ripple

$\epsilon \rightarrow$ controls passband ripple

$C_n \rightarrow$ Chebychev polynomial of order n with the properties

$0 \leq \frac{\omega}{\omega_c} \leq 1$, then $0 \leq C_n^2\left(\frac{\omega}{\omega_c}\right) \leq 1$

$\frac{\omega}{\omega_c} > 1$, then $C_n^2\left(\frac{\omega}{\omega_c}\right) > 1$

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Starting with $C_0\left(\frac{\omega}{\omega_c}\right)=1$ and $C_1\left(\frac{\omega}{\omega_c}\right)=\frac{\omega}{\omega_c}$ we can obtain C polynomials:

$$C_n = 2\left(\frac{\omega}{\omega_c}\right)C_{n-1} - C_{n-2}$$

$$C_n\left(\frac{\omega}{\omega_c}\right) = \begin{cases} \cos\left[n\cos^{-1}\left(\frac{\omega}{\omega_c}\right)\right] & 0 \le \frac{\omega}{\omega_c} \le 1 \\ \cosh\left[n\cosh^{-1}\left(\frac{\omega}{\omega_c}\right)\right] & \frac{\omega}{\omega_c} > 1 \end{cases}$$

Compared to Butterworth; stopband attenuation $\sim \frac{\omega}{\omega_c}=1$ is steeper.

$$n \ge \frac{\cosh^{-1}\left[(10^{0.1A_s}-1)/(10^{0.1A_p}-1)\right]^{\frac{1}{2}}}{\cosh^{-1}(\omega_s/\omega_c)}$$

Normalising frequency assumes $\omega_c = 1$ rad/sec, it is shown that the Chebychev roots lie on an ellipse governed by the ripple.

$$\varepsilon = (10^{0.1A_p}-1)^{\frac{1}{2}}$$, similar to before.

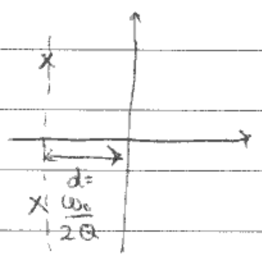
Chebychev filters are efficient at filtering out frequencies components, at the expense of pass-band ripple.

Biquadratic function

Consider this transfer function (where both numerator & denominator are quadratic)

$$H(s) = \frac{n_2 s^2 + n_1 s + n_0}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

ω_0 & Q determine the location of the poles.



if $Q \le 0.5$, negative real axis

if $Q > 0.5$, poles are complex conjugates.

Notice that attenuation statistics are garbage, because these are only second order filters.

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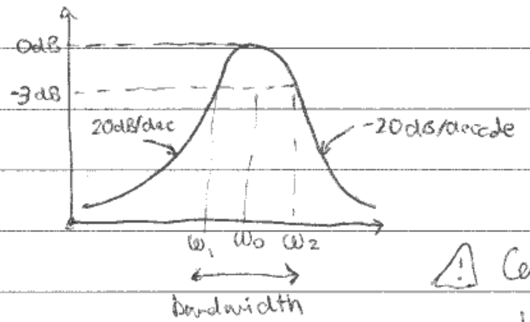
e.g. Butterworth filters are critically damped

- if $n_0 = n_1 = 0$, we have a High pass filter

$$H(s) = \frac{H_0 s^2}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

's' terms influence a 40dB/decade rise in a $\log(\omega)$ vs $|H(j\omega)|$ plot.

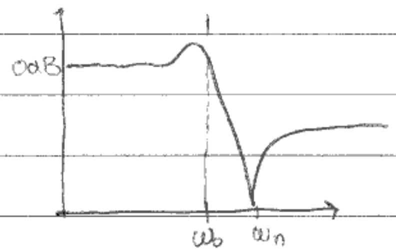
- $n_0 = n_2 = 0$, we have a Band Pass filter



$$H(s) = \frac{H_0\left(\frac{\omega_0}{Q}\right)s}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

⚠ Cascade several stages of these together to obtain a true band-pass filter.

- $n_1 = 0$ gives a Notch filter



$$H(s) = n_2 \frac{s^2 + \omega_n^2}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

- Finally consider the case where n_1, n_2 and n_0 are non-zero. This will give an All-Pass filter. Zeroes and poles are mirror images about the $j\omega$ axis. Used to modify the phase shift (only) of a system, leaving magnitude unchanged.

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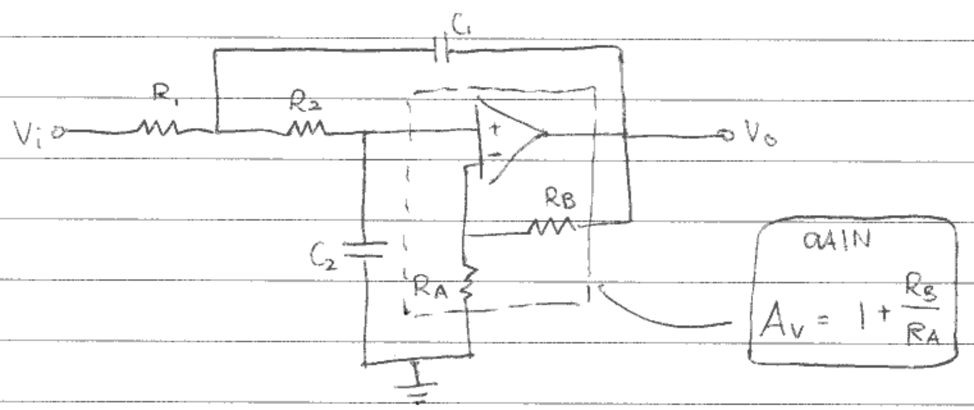
Single Amp Biquad Filter



Performing nodal analysis we get:

$$H(s) = \frac{V_o}{V_i} = \frac{A_v}{R_1 R_2 C_1 C_2 s^2 + \{C_2(R_1+R_2) + R_1 C_1 (1-A_v)\} s + 1}$$

Which is identifiable as a low-pass filter ($n_1 = n_2 = 0$ in the biquad transfer function)

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \qquad Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 C_1 (1-A_v) + C_2 (R_1 + R_2)}$$

We can simplify to

$$H(s) = \frac{A_v}{R^2 C^2 s^2 + (3-A_v) RC s + 1}, \quad \underbrace{\omega_0 = \frac{1}{RC}}_{\text{by RC time constant}} \quad \& \quad \underbrace{Q = \frac{1}{3-A_v}}_{\text{by gain of amplifier.}}$$

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Transformations

i, Lowpass to lowpass

$$s = \frac{s'}{\omega_0} = \frac{j\omega}{\omega_0} = j\Omega$$

$$\Omega = \frac{\omega}{\omega_0}.$$

Given that passband is at $\omega_p = \omega_0$ & stopband is at ω_s. All we need to do is de-normalise the transfer function.

ii, Lowpass to highpass.

Invert the transfer function, $s = \frac{\omega_0}{s'}$, so $\Omega = \frac{-\omega_0}{\omega}$.

Passband is always higher than stopband (clearly).

iii, Low-pass to band-pass

$$s = \frac{1}{B} \cdot \frac{s'^2 + \omega_0^2}{s'} = \frac{1}{B/\omega_0} \cdot \frac{(s'/\omega_0)^2 + 1}{(s'/\omega_0)}.$$

You'll need to choose some parameters

- s : LPF frequency variable (normed)
- s' : BPF frequency variable
- ω_0 : centre frequency
- B : bandwidth of BPF.

When $s=0$, $s' = \pm j\omega_0$. Expect 4 different frequencies with the same attenuation:

$$\omega_2 = \frac{B\Omega}{2} + \sqrt{\omega_0^2 + \left(\frac{B\Omega}{2}\right)^2}$$

$$\omega_1 = -\frac{B\Omega}{2} + \sqrt{\omega_0^2 + \left(\frac{B\Omega}{2}\right)^2}$$

And $\omega_1 \omega_2 = \omega_0^2$ and $\omega_2 - \omega_1 = B\Omega$.

We will need to tolerate up to A_p dB of attenuation.

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i/ $\omega_0 = \sqrt{\omega_{p2}\,\omega_{p1}}$

ii/ $B = \omega_{p2} - \omega_{p1}$

iii/ If $\frac{\omega_0^2}{\omega_{s1}} \leq \omega_{s2}$, choose $\omega_{s1} = \omega_{s1}$ & $\omega_{s2} = \omega_0^2/\omega_{s1}$

but if $\frac{\omega_0^2}{\omega_{s1}} > \omega_{s2}$, choose $\omega_{s2} = \omega_{s2}$ & $\omega_{s1} = \omega_0^2/\omega_{s2}$.

iv/ determine Ω_s.

v/ from attenuation values and Ω_s we can determine order of the filter and the Lowpass prototype.

vi/ Knowing ω_0 & B we can find the BPF.

iv/ Low-pass to band-stop

$$s = \frac{Bs'}{s'^2 + \omega_0^2} \rightarrow$$ same as BPF but upside down.

Summary: We normally implement S+K filters with non inverting amplifiers. it is much more complex to use an inverting amplifier. swapping capacitors & resistors turns an LPF into a HPF. also with inverting amps, input impedance is finite, and gain requirements dictate you may only use them for low frequency.

Infinite Gain Multiple Feedback Circuits

$$\frac{V_o}{V_i} = \frac{-y_1\, y_4}{y_5(Y_1+Y_2+Y_3+Y_4)}$$

are our y's capacitors or resistors

Given Q, $\omega_0 = 2\pi f_0$ and H_0 we can choose $C_2 = C$ and $C_5 = kC_2$ then calculate R_4, R_1 & R_3.

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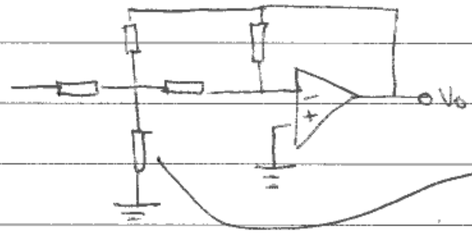

are these resistors or capacitors?
choose an appropriate ratio.

Sek vs **Multiple Feedback**

Sek

- small spread in R, L, C values
- Q & ω_0 sensitive to R, L, C variations
- % change in Q, ω_0 is many times the % change in element values.
- H_0 & Q dependent of one another

Multiple Feedback

- large spread, increases with gain
- Q & ω_0 insensitive to network elements.
- % change in Q & ω_0 is less than (at worst equal to) the magnitude of the % change in element values
- H_0 & Q independant.